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Inducing the μ and the $B\mu$ Term by the Radion and the 5d Chern-Simons Term

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Abstract

In 5-dimensional models with gauge-Higgs unification, the F -term vacuum expectation value of the radion provides, in close analogy to the Giudice-Masiero mechanism, a natural source for the μ and $B\mu$ term. Both the leading order gauge theory lagrangian and the supersymmetric Chern-Simons term contain couplings to the radion superfield which can be used for this purpose. We analyse the basic features of this mechanism for μ term generation and provide an explicit example, based on a variation of the SU(6) gauge-Higgs unification model of Burdman and Nomura. This construction contains all the relevant features used in our generic analysis. More generally, we expect our mechanism to be relevant to many of the recently discussed orbifold GUT models derived from heterotic string theory. This provides an interesting way of testing high-scale physics via Higgs mass patterns accessible at the LHC.

1 Introduction

The generation of a μ and $B\mu$ term in the Higgs sector of the supersymmetric standard model is one of the critical issues in low-energy supersymmetry. While the μ term alone is responsible for Higgsino masses, both terms play a central role in realizing an appropriate scalar potential in the Higgs sector, ensuring the spontaneous breaking of the electroweak gauge symmetry. Since the μ term respects supersymmetry, one might also formulate the $\mu/B\mu$ term problem by asking why this term, which would naturally be either very large or exactly zero, happens to be of the same order of magnitude as the soft supersymmetry-breaking $B\mu$ term [1].

The two most popular solutions to this problem are provided by the Giudice-Masiero mechanism [2] and the next-to-minimal supersymmetric standard model [3]. In the latter, the scale of the μ term is set by the vacuum expectation value of the scalar component of an extra uncharged chiral superfield. By contrast, in the former the μ term arises from a term in the Kähler potential, which mimics a μ term in the superpotential after the non-zero F term of the spurion superfield has absorbed part of the superspace integrations. Many variants of these mechanisms as well as other approaches to the problem have since been considered (see [4] for some recent examples).

In the present paper, we investigate 5-dimensional models with gauge-Higgs unification [5], where the $\mu/B\mu$ term problem is solved naturally in a way which is very similar to the Giudice-Masiero mechanism. Both these terms as well as the gaugino mass term and some of the soft scalar masses are generated at the high scale in the interplay of the F term of the radion superfield and the chiral compensator of $\mathcal{N} = 1$ supergravity with the quadratic gauge theory lagrangian [6] (see also [7]). We point out that the resulting high-scale relations are changed significantly by the 5d Chern-Simons term which, in particular, induces a non-trivial Higgs scalar potential even in the absence of an F term of the chiral compensator.

At the more fundamental level, our motivation for this work is twofold: On the one hand, orbifold-GUTs [8] are arguably *the* modern framework for grand unification. Within this framework, gauge-Higgs unification receives a strong motivation from the requirement of a large top Yukawa coupling. Furthermore, it is natural that both the radion superfield [9] and (after radion stabilization) also the chiral compensator develop an F -term vacuum expectation value. Thus, all ingredients for our mechanism are naturally present and the required terms in the supersymmetric Higgs sector arise without any further model building assumptions.

On the other hand, heterotic orbifold model building has recently produced some of the most successful string-theoretic realizations of the supersymmetric standard model [10] (for earlier related work see [11]). From this perspective, the existence of an intermediate energy scale (one or two orders of magnitude below the string scale), at which the world appears to be 5-dimensional, is also well-motivated [12]. It provides one of the few potential solutions to the string-scale/GUT-scale problem. Furthermore, gauge-Higgs unification is again a natural ingredient in all constructions where the Higgs fields come from the untwisted sector, which is indeed the case in many concrete exam-

ples.

The presence of a μ term in 5d models with gauge-Higgs unification has been noticed early on [13].¹ The simultaneous generation of a $B\mu$ term by the F -term vev of the chiral compensator, leading to an interesting relations between μ term, $B\mu$ term and non-holomorphic soft Higgs masses, has been pointed out in [6]. This relation is maintained in the presence of a 5d Chern-Simons term, which however changes the relation with the gaugino masses. As we already mentioned, the Chern-Simons term is crucial in situations where the F term of the chiral compensator is small. Although such a term is generically present in 5d supersymmetric gauge theories [14] (see also [15]), it affects low-energy phenomenology only if some of the scalars of the 5d gauge multiplet develop large vacuum expectation values [16]. This is, however, very well motivated in stringy realizations of our scenario, where more than 5 dimensions are originally present. In most cases, some of these extra compact dimensions support non-zero Wilson lines which can, from a 5d perspective, play the role of the required scalar vacuum expectation value. In such situations, the supersymmetric Chern-Simons term is parametrically as important for low-energy phenomenology as the quadratic lagrangian.

We finally note that a detailed phenomenological analysis of the proposal advocated in the present paper has subsequently appeared in [17]. In addition to demonstrating the phenomenological viability of our setting, this work was essential for bringing an earlier, partially incorrect version of this paper in its present form. We will comment on the earlier proposal, its problems and their possible resolutions in more detail below.²

Our paper is organized as follows: We begin in Sect. 2 with the discussion of an abelian toy model which shows, in a very direct and transparent way, how the quadratic gauge theory lagrangian and the Chern-Simons term induce, in their interplay with the radion superfield, terms that are structurally similar to the μ and $B\mu$ term and soft supersymmetry breaking masses for the ‘Higgs field’.

In Sect. 3, we extend our analysis to the non-abelian case, providing in particular a superfield expression for the non-abelian supersymmetric Chern-Simons term. The derivation of this term, which we consider to be a very interesting by-product of our investigation, is described in more detail in the Appendix. Applying our formulae to a $U(6) = SU(6) \times U(1)$ model, where the possibility of gauge-Higgs unification is particularly apparent from the decomposition $\mathbf{35} = \mathbf{24} + \mathbf{5} + \bar{\mathbf{5}} + \mathbf{1}$ of the adjoint [13], we identify the terms involving the two Higgs superfields, the radion and the chiral compensator.

We use our previous results to calculate, in Sect. 4, μ and $B\mu$ term, as well as soft Higgs scalar masses and gaugino masses. As an interesting observation we note that, in the absence of the Chern-Simons term and of an F term of the chiral compensator, μ term and soft scalar masses conspire to ensure an exactly flat scalar potential in the Higgs sector. However, once the radion is stabilized, a chiral compensator F term generically develops and this flatness is lifted.

¹ An alternative proposal in closely related string-theoretic models appears in the last paper of Ref. [10].

² We are indebted to Felix Brümmer pointing out the problems of the original setting.

In Sect. 5, we give the complete expressions for the μ term and the soft parameters of the gauge-Higgs sector, including the effects of the Chern-Simons term and chiral compensator. We then briefly discuss the viability of this high-scale input for low-energy phenomenology after the renormalization group running down to the electroweak scale. We also comment on the influence of the squark masses and trilinear terms on this running and on the partially model-dependent high-scale origin of these terms (especially in the top quark sector) in our 5d gauge-Higgs unification scenario.

Finally, we provide in Sect. 6 an explicit phenomenologically viable construction that has all the qualitative features which we used in our previous discussion. Our model is closely related to a 5d model for gauge-Higgs unification by Burdman and Nomura [13]. We obtain our model by lifting this previous construction to 6 dimensions, where the compact space has the topology of a pillow case, and taking a different 5d limit of this geometry. In this way the non-zero 5d vev of the scalar component of the gauge multiplet is automatically enforced. The rather intricate realization of matter fields and Yukawa couplings can essentially be copied from the construction of Burdman and Nomura.

Our summary and conclusions are given in Sect. 7.

2 The basic mechanism in an abelian toy model

The supersymmetric 5d U(1) gauge theory has a well-known description in terms of a 4d real superfield V and a chiral superfield $\Phi = \Sigma + iA_5 + \dots$, both depending on the extra parameter x^5 . Using this language, the quadratic 5d lagrangian reads [18, 19]

$$\mathcal{L}_2 = \frac{1}{4g_5^2} \left[\int d^2\theta W^2 + \text{h.c.} + \int d^4\theta \left(2\partial_5 V - (\Phi + \bar{\Phi}) \right)^2 \right], \quad (1)$$

where W is the supersymmetric field strength defined in terms of V . The supersymmetric Chern-Simons term which will in general be present in this theory takes the form [19]³

$$\begin{aligned} \mathcal{L}_{cs} = c & \left[\int d^2\theta \Phi W^2 + \text{h.c.} \right. \\ & + \frac{2}{3} \int d^4\theta (\partial_5 V D_\alpha V - V D_\alpha \partial_5 V) W^\alpha + \text{h.c.} \\ & \left. - \frac{1}{6} \int d^4\theta \left(2\partial_5 V - (\Phi + \bar{\Phi}) \right)^3 \right]. \end{aligned} \quad (2)$$

We are interested in the 4d effective field theory obtained after S^1 compactification of the above model, in particular in the couplings to the radion superfield. The following discussion can be viewed as a mild generalization of [20] (because of the Chern-Simons term) or as a significantly simplified version of the derivation of related formulae in [21].

The relevant 4d lagrangian is found by simply dropping all terms involving x^5 derivatives, replacing V and Φ by their (x^5 -independent) zero modes, and integrating the result

³ Note that we find a different sign for the second term than is reported in [19].

over x^5 . In the rigid case, the latter amounts to a multiplication by $L = 2\pi R$. By contrast, in the case where the original model is coupled to 5d supergravity, this multiplicative factor has to be replaced by the radion superfield T (or \bar{T}) in the holomorphic (antiholomorphic) terms of Eqs. (1) and (2) and by $(T + \bar{T})/2$ in the $d^4\theta$ terms. Here the 4d chiral superfield T is normalized such that

$$T = L + iB_5, \quad (3)$$

where B_M ($M = 0 \dots 3, 5$) is the graviphoton of the 5d supergravity multiplet. Its pure-derivative coupling in the component action enforces the use of the combination $T + \bar{T}$ in the $d^4\theta$ terms in Eqs. (1) and (2).

However, this is not the only way in which T enters the 4d effective theory. From the fact that Φ contains the gauge field component A_5 , and A_5 covariantizes the derivative operator $\partial/\partial x^5$, it follows that the whole superfield has to scale as the inverse size of the compact dimension. Thus, we have to perform the replacements

$$\Phi \rightarrow \frac{L_0}{T} \Phi \quad \text{and} \quad \Phi \rightarrow \frac{2L_0}{T + \bar{T}} \Phi \quad (4)$$

in the $d^2\theta$ and $d^4\theta$ terms above. Here we have introduced an arbitrary constant L_0 with the dimension of length to insure that the new superfield Φ has the dimension of mass.

To summarize, the 4d low-energy lagrangian follows from Eqs. (1) and (2) after suppressing any x^5 dependence, multiplying the appropriate terms by T , \bar{T} or $(T + \bar{T})/2$, and performing the redefinition of Eq. (4). The results are

$$\mathcal{L}_{2,4d} = \frac{1}{4g_5^2} \left[\int d^2\theta \, T W^2 + \text{h.c.} + 2L_0^2 \int d^4\theta \frac{(\Phi + \bar{\Phi})^2}{T + \bar{T}} \right] \quad (5)$$

and

$$\mathcal{L}_{cs,4d} = c \left[L_0 \int d^2\theta \, \Phi W^2 + \text{h.c.} + \frac{4L_0^3}{6} \int d^4\theta \frac{(\Phi + \bar{\Phi})^3}{(T + \bar{T})^2} \right]. \quad (6)$$

To check that the T dependence obtained in this intuitive approach is indeed correct, one can work out the component form of the above superfield expressions and match it (with appropriate field redefinitions and keeping track of all factors g_{55}) to the 5d component action [22].

Our main point concerning the generation of certain MSSM operators can now easily be made. Recall that we want to think of V as containing the Standard model gauge multiplet and of Φ as the Higgs superfield.⁴ If the radion auxiliary field F_T develops a non-zero expectation value, it is immediately clear that the superspace integrals in Eq. (5) induce operators

$$\sim F_T W^2 \Big|_1, \quad \sim |F_T|^2 \Phi^2 \Big|_1, \quad \sim \bar{F}_T \Phi^2 \Big|_{\theta^2}, \quad \sim |F_T|^2 \Phi \bar{\Phi} \Big|_1 \quad \text{and} \quad \sim \bar{F}_T \Phi \bar{\Phi} \Big|_{\theta^2}. \quad (7)$$

⁴ Of course, in this simple U(1) toy model Φ is not charged and the second Higgs multiplet is missing, but that is irrelevant for now.

The first of them provides gaugino masses, which is often referred to as radion mediation [9]. The second, which clearly has the structure of the MSSM μ term, provides Higgsino masses.⁵ Furthermore, both the second and the remaining operators in Eq. (7) contribute to the scalar potential, thereby apparently inducing a $B\mu$ term and soft scalar masses in the Higgs sector. However, a more careful analysis of Eq. (5) reveals that all these contributions exactly cancel and the scalar potential remains flat. (This fact, which can also be understood from a structural perspective [23], remains true in the non-abelian case.)

To lift the flatness of the potential and to induce a non-zero $B\mu$ term and soft scalar masses in the present framework, the effect of the chiral compensator of $\mathcal{N} = 1$ supergravity, $\varphi = 1 + F_\varphi \theta^2$, has to be taken into account. More specifically, a factor $\varphi \bar{\varphi}$ has to be included the last term in Eq. (5). If F_φ develops a non-zero vacuum expectation value, operators analogous to those displayed in Eq. (7) (but with one or both of the factors F_T and \bar{F}_T replaced by F_φ and \bar{F}_φ) are induced. The total scalar potential loses its flatness, which can be described by a non-vanishing $B\mu$ and soft scalar mass terms.

If the lowest component of Φ develops a vacuum expectation value, then the Chern-Simons lagrangian of Eq. (6) corrects the quadratic order lagrangian of Eq. (5). Moreover, if $c = \mathcal{O}(1)$ and $\langle \Phi \rangle \sim 1/g_5^2$ (both of which are natural values, as will become clear in the following), these contributions are not parametrically suppressed relative to those of Eq. (5). Thus, gaugino masses, μ and $B\mu$ term, and the Higgs sector soft scalar masses are induced on the basis of the fundamental lagrangian of Eqs. (1) and (2) after coupling it to supergravity and allowing for vacuum expectation values of Φ , F_T and F_φ . As we will explain in more detail below, in higher-dimensional unified models an interesting and realistic phenomenology can emerge on the basis of this very generic mechanism.

3 Non-abelian generalization

The $\mathcal{N} = 1$ superfield action of the 5d non-abelian gauge theory [18,19] can be given in a manifestly super-gauge-invariant form using the super-gauge-covariant x^5 derivative [24]

$$\nabla_5 = \partial_5 + \Phi. \quad (8)$$

It reads

$$\mathcal{L}_2 = \frac{1}{2g_5^2} \text{tr} \left[\int d^2\theta W^2 + \text{h.c.} + \int d^4\theta \left(e^{-2V} \nabla_5 e^{2V} \right)^2 \right], \quad (9)$$

where the action of Φ on e^{2V} follows from the standard gauge transformation properties of e^{2V} , i.e.,

$$\nabla_5 e^{2V} = \partial_5 e^{2V} - \Phi^\dagger e^{2V} - e^{2V} \Phi. \quad (10)$$

⁵ This can be understood from a slightly different perspective as follows: Non-vanishing F_T is the 4d manifestation of an $SU(2)_R$ symmetry twist of in the 5d background. The latter induces gaugino masses and, since the Higgsinos are 5d gauginos in the present setting, non-vanishing Higgsino masses are also induced [13].

For the non-abelian supersymmetric Chern-Simons term we have, unfortunately, not been able to derive an equally elegant superfield formula. However, sacrificing manifest super gauge invariance by restricting ourselves to Wess-Zumino gauge, the following expression can be derived [22] (see Appendix):

$$\begin{aligned}
\mathcal{L}_{cs} = & c \operatorname{tr} \left[\int d^2\theta \, \Phi W^2 + \text{h.c.} \right. \\
& + \frac{1}{3} \int d^4\theta \, (\{\partial_5 V, D_\alpha V\} - \{V, D_\alpha \partial_5 V\}) W^\alpha + \text{h.c.} \\
& - \frac{1}{12} \int d^4\theta \, (\{\partial_5 V, D_\alpha V\} - \{V, D_\alpha \partial_5 V\}) W_{(2)}^\alpha + \text{h.c.} \\
& \left. - \frac{1}{6} \int d^4\theta \, (e^{-2V} \nabla_5 e^{2V})^3 \right]. \tag{11}
\end{aligned}$$

Here curly brackets are used for anticommutators and $W_{(2)}^\alpha$ represents the part of W^α which is quadratic in V (recall that, in Wess-Zumino gauge, W is the sum of a linear and quadratic piece in V).

Starting from Eqs. (9) and (11), which are the non-abelian generalizations of Eqs. (1) and (2), the coupling of the radion superfield to the zero modes of the compactified theory can be derived in complete analogy to Sect. 2. To recapitulate, one simply has to suppress any x^5 dependence, multiply the appropriate terms by T , \bar{T} or $(T + \bar{T})/2$, and perform a redefinition analogous to that of Eq. (4). The results are

$$\mathcal{L}_{2,4d} = \frac{1}{2g_5^2} \operatorname{tr} \left[\int d^2\theta \, T W^2 + \text{h.c.} + 2L_0^2 \int d^4\theta \, \frac{(\Phi + \bar{\Phi})^2}{T + \bar{T}} \right] \tag{12}$$

and

$$\mathcal{L}_{cs,4d} = c \operatorname{tr} \left[L_0 \int d^2\theta \, \Phi W^2 + \text{h.c.} + \frac{4L_0^3}{6} \int d^4\theta \, \frac{(\Phi + \bar{\Phi})^3}{(T + \bar{T})^2} \right]. \tag{13}$$

Clearly, this could have also been obtained by starting from Eqs. (5) and (6), promoting the superfields V and Φ to appropriate matrices and introducing the corresponding trace operations. In this sense, our above discussion of the 5d superfield expression for the non-abelian Chern-Simons term is included merely for completeness (and possible other applications). The phenomenology-oriented analysis following from now on is based entirely on Eqs. (12) and (13), which are straightforward generalizations of Eqs. (5) and (6).

We can now be more specific about how we envisage the μ and $B\mu$ term generation to proceed in models of this type. To be concrete, let V and Φ take values in the Lie algebra of the GUT gauge group $U(6) = SU(6) \times U(1)$. Furthermore, let the theory be compactified to 4d on an interval such that $SU(6)$ is broken to $SU(5) \times U(1)'$ and the $U(1)$ is completely broken. In the corresponding decomposition of the adjoint representation,

$$\mathbf{35} = \mathbf{24} + \mathbf{5} + \bar{\mathbf{5}} + \mathbf{1}, \tag{14}$$

we find, as parts of the superfield Φ , the Higgs multiplets H_u and H_d in the $\mathbf{5}$ and $\bar{\mathbf{5}}$ of $SU(5)$. The further breaking of $SU(5)$ to the standard model gauge group, which could for example also be realized by boundary conditions, is not important at the moment.

Thus, the second term of Eq. (12) gives rise to the following contribution to the 4d Higgs lagrangian:

$$\mathcal{L}_{2,4d} \supset \frac{1}{g_4^2} \int d^4\theta \frac{2L_0\varphi\bar{\varphi}}{T+\bar{T}} (H_u + \bar{H}_d)(H_d + \bar{H}_u). \quad (15)$$

Furthermore, if Φ develops a vev $\langle\Phi\rangle = v\mathbf{1}$, consistent with the assumed boundary-breaking of the $U(1)^6$, the second term of Eq. (13) gives rise to the following correction to this lagrangian (up to quadratic order):

$$\mathcal{L}_{cs,4d} \supset 2cL_0v \int d^4\theta \frac{(2L_0)^2\varphi\bar{\varphi}}{(T+\bar{T})^2} (H_u + \bar{H}_d)(H_d + \bar{H}_u). \quad (16)$$

Here we have assumed that, with the exception of H_u and H_d , all the zero-mode components of the chiral adjoint Φ have been eliminated by orbifolding (or acquired a large mass in another way). Note also that, since we are not interested in the dynamics of T and φ at the moment, we have suppressed the constant term $\sim v^3$ in Eq. (16). A term $\sim v^2$, which would have to be linear in H_u and H_d , does obviously not arise for group theoretic reasons.

In a vacuum where T and φ develop non-zero F terms, Eqs. (15) and (16) provide, in addition to the kinetic terms for the Higgs multiplets, μ term, $B\mu$ term and soft scalar masses in the Higgs sector. The relevant operators are analogous to those given explicitly in the case of our abelian toy model in Eq. (7) of the previous section. In addition, the first terms of both Eq. (12) and (13) contribute to the standard model gauge kinetic term and to the corresponding gaugino masses. We devote the following two sections to the discussion of the resulting SUSY breaking pattern.

⁶ In an earlier version of this paper, a Φ -vev $\sim \text{diag}(1, 1, 1, 1, 1, -5)$ inside the adjoint of $SU(6)$ was assumed. This is inconsistent with an orbifold breaking of $SU(6)$ to $SU(5) \times U(1)'$. The desired breaking by boundary conditions can nevertheless be realized, e.g. by introducing a brane localized adjoint superfield and giving it a large vev $\sim \text{diag}(1, 1, 1, 1, 1, -5)$. However, the bulk vev of Φ induces a bulk mass for the $\mathbf{5}$ and $\bar{\mathbf{5}}$ Higgs fields. This is easy to see since the gauge symmetry is broken in the 5d bulk. Hence the ‘broken’ A_5 components, which form some of the Higgs scalars, become massive in 5d. Equivalently, when thinking at the zero-mode level of a corresponding S^1 compactification, this mass term must be present since the $\mathbf{5}$ and $\bar{\mathbf{5}}$ chiral multiplets become part of the massive vector multiplet. On an interval with boundary-breaking, massless 4d fields in these representations nevertheless survive since only a certain linear combination of the bulk and brane $\mathbf{5}$ and $\bar{\mathbf{5}}$ fields is ‘eaten’ by the vector multiplets which become massive in the breaking of $SU(6)$ to $SU(5) \times U(1)'$. However, these massless Higgs fields now have a non-trivial bulk profile because of their bulk mass. This profile depends on the size of the Φ -vev and affects both the calculation of soft terms and of Yukawa couplings, thereby significantly complicating the subsequent analysis. This set of problems as well as its resolution by simply using $U(6)$ instead of $SU(6)$ was pointed out to us by Felix Brümmer (see also [17]).

We also note that $U(6)$ is, of course, a product gauge group allowing for independent coefficients of the $SU(6)$ - and $U(1)$ -kinetic terms as well as of the CS terms of $SU(6)$, $U(1)$ and of the mixed CS terms (see e.g. [25]). Since, given the above Φ -vev, only the mixed CS term is relevant for our analysis, we do not complicate our notation by making all those independent coefficients explicit.

4 Calculating the μ and $B\mu$ term and the Higgs-sector soft scalar masses

To begin, we ignore the possible Chern-Simons term and focus on the phenomenological implications of Eq. (15) and the first term of Eq. (12). We assume the existence of a (meta-)stable almost-Minkowski vacuum in which $\text{Re}T = L_0$ and both F_T and F_φ have non-zero values. Using the chiral compensator approach to supergravity, the scalar potential in the Higgs sector (with canonical 4d field normalization) is easily obtained: We simply have to integrate out the auxiliary-field vectors F_{H_u} and F_{H_d} on the basis of Eq. (15) while treating T , F_T and F_φ as fixed external sources. The result reads

$$\mathcal{L}_{4,can.} \supset - \left(|F_\varphi|^2 - \frac{F_\varphi \bar{F}_T + \bar{F}_\varphi F_T}{T + \bar{T}} \right) (H_u + \bar{H}_d)(H_d + \bar{H}_u). \quad (17)$$

We emphasize that, in contrast to the last section, in this and the following equations H_u and H_d are the scalar components of the corresponding superfields and their normalization has been modified to make the 4d kinetic term canonical. The corresponding Higgsino mass term can be directly read off from Eq. (15):

$$\mathcal{L}_{4,can.} \supset - \left(\bar{F}_\varphi - \frac{\bar{F}_T}{T + \bar{T}} \right) \lambda_u \lambda_d + \text{h.c.} \quad (18)$$

where λ_u and λ_d are two-component Weyl spinors. This determines the value of the μ parameter, which is conventionally defined as the coefficient of the Higgsino bilinear:

$$\mu = \bar{F}_\varphi - \frac{\bar{F}_T}{T + \bar{T}}. \quad (19)$$

Similarly to the gaugino mass

$$m_{1/2} = \frac{\bar{F}_T}{T + \bar{T}}, \quad (20)$$

a non-zero μ parameter arises as a consequence of F_T , even if F_φ vanishes.

Furthermore, if the Higgs scalar potential is parameterized by (see e.g. [26])

$$\mathcal{L}_{4,can.} \supset -(|\mu|^2 + m_{H_u}^2)|H_u|^2 - (|\mu|^2 + m_{H_d}^2)|H_d|^2 - (B\mu)H_u H_d + \text{h.c.} + \text{quart. terms}, \quad (21)$$

we read off from Eq. (17) that $B\mu$, m_{H_u} and m_{H_d} are given by (see also [6])

$$B\mu = |\mu|^2 + m_{H_u}^2 = |\mu|^2 + m_{H_d}^2 = |F_\varphi|^2 - \frac{F_\varphi \bar{F}_T + \bar{F}_\varphi F_T}{T + \bar{T}}. \quad (22)$$

In contrast to the μ parameter, these scalar mass parameters vanish if $F_\varphi = 0$. This is a result of the very specific generalized no-scale structure of the superfield expression in Eq. (12). In terms of the conventional parameterization of the component lagrangian with soft terms, it implies a somewhat surprising exact cancellation between $|\mu|^2$ and $m_{H_u}^2$ as well as between $|\mu|^2$ and $m_{H_d}^2$ in Eq. (22). Clearly, the phenomenological implications

of the above formulae crucially depend on the values of F_T and F_φ (especially on their relative size), on which we now briefly comment.

At the tree level, the compactification of 5d supergravity on S^1/Z_2 or $S^1/(Z_2 \times Z'_2)$ gives rise to a Kähler potential of no-scale type for the radion,

$$K_0(T, \bar{T}) = -3 \ln(T + \bar{T}) . \quad (23)$$

An effective constant superpotential can be introduced if the boundary conditions at the two ends of the interval preserve different $\mathcal{N} = 1$ subalgebras of the original $\mathcal{N} = 2$ SUSY. (Alternatively, the same effect can arise as a result of some non-perturbative boundary effect, such as brane gaugino condensation.) In the resulting no-scale model, supersymmetry is broken by F_T , but T remains a flat direction. At the same time, F_φ remains exactly zero. For our purposes, this approximation (in the case that this is a reasonable approximation to the physical vacuum) is insufficient since, as already mentioned in Sect. 2, the Higgs sector scalar potential remains exactly flat in this case.

Thus, we have to take the stabilization of the radion T seriously from the very beginning and to determine F_T and F_φ in the context of a stabilized vacuum. It is well-known that F_φ is generically non-zero in such situations (implying, in our context, that a Higgs sector scalar potential will be generated).

Starting from the no-scale situation described above, stabilization of T can arise as a result of either Kähler corrections or T -dependent superpotential terms. To be as generic as possible, we assume a model where, on the basis of a corrected Kähler potential and superpotential,

$$K(T, \bar{T}) = K_0(T, \bar{T}) + \Delta K(T, \bar{T}) \quad \text{and} \quad W(T) , \quad (24)$$

a (meta-)stable almost-Minkowski vacuum is produced (see e.g. [27, 28]). The equations of motion for F_T and F_φ (and thus their vacuum values) can be obtained on the basis of the flat-space superfield lagrangian

$$\int d^4\theta \varphi \bar{\varphi} \Omega(T, \bar{T}) + \int d^2\theta \varphi^3 W(T) + \text{h.c.} , \quad (25)$$

where $\Omega = -3 \exp(-K/3)$ is the so-called ‘superspace kinetic energy’ [29].

For the purpose of this paper, we do not want to specify a stabilization mechanism for T and extremize Eq. (25) explicitly. Instead, we restrict ourselves to deriving a simple relation between the F terms of the radion and the chiral compensator. This can be achieved rather easily: First, assume that Eq. (25) possesses a SUSY-breaking minimum with vanishing cosmological constant. In this minimum, W takes some vacuum expectation value W_0 . We now go to a different Kähler-Weyl frame, defined by the requirement that the superpotential W' in this frame is constant, $W' = W_0$. Such a change of frames can be viewed as a redefinition of the chiral compensator. The new chiral compensator φ' is defined in terms of T and φ by

$$W(T) \varphi^3 = W' \varphi'^3 . \quad (26)$$

In this new frame, $F_{\varphi'} = 0$, which is an immediate consequence of vanishing vacuum energy and constant superpotential (see e.g. [30]). Thus,

$$\varphi = \varphi' \cdot \left(\frac{W(T)}{W_0} \right)^{-1/3} = 1 \cdot \left(1 + \frac{W_T F_T \theta^2}{W_0} \right)^{-1/3}. \quad (27)$$

To lighten notation, we can now suppress the index ‘0’ of W and simply conclude that

$$F_\varphi = -\frac{W_T}{3W} F_T \quad (28)$$

in the physical vacuum. This formula allows for a simple evaluation of the previously derived supersymmetric and SUSY-breaking Higgs mass terms and their relation to gaugino masses in any concrete model of radius stabilization. Note that, for a generic function $W(T)$, we expect $F_\varphi \sim F_T/T$ on dimensional grounds. This relation is also found in the specific model of [27]. The SUSY-breaking effects of F_φ and F_T are then parametrically equally important.

5 Including the effect of the Chern-Simons term and some phenomenological consequences

We now repeat the analysis of the previous section on the basis of the complete lagrangian of Eqs. (15) and (16). Integrating out F_{H_u} and F_{H_d} , the following (canonically normalized) scalar potential arises:

$$\mathcal{L}_{4,can.} \supset - \left[|F_\varphi|^2 - \frac{(F_\varphi \bar{F}_T + \text{h.c.})}{T + \bar{T}} \frac{1 + 2c'}{1 + c'} + \frac{|F_T|^2}{(T + \bar{T})^2} \frac{2c'^2}{(1 + c')^2} \right] (H_u + \bar{H}_d)(H_d + \bar{H}_u), \quad (29)$$

where

$$c' = 2cv g_5^2. \quad (30)$$

Note that the no-scale argument ensuring the complete flatness of the scalar potential in the absence of F_φ has broken down. The reason is as follows: While the Chern-Simons term by itself respects the generalized no-scale structure, the presence of a fixed vev v breaks this structure. For this it is crucial that the vev is truly fixed in the sense that no corresponding fluctuations are allowed - a situation which indeed arises in certain orbifold models (see below).

Similarly, the Higgsino mass term, Eq. (18), is now replaced by an analogous expression following from Eqs. (15) and (16):

$$\mathcal{L}_{4,can.} \supset \left(\bar{F}_\varphi - \frac{\bar{F}_T}{T + \bar{T}} \frac{1 + 2c'}{1 + c'} \right) \lambda_u \lambda_d + \text{h.c.} \quad (31)$$

The gaugino mass is also affected by the Chern-Simons term. Although F_Φ does not develop a vacuum expectation value, the first term in Eq. (13) affects the normalization

of the gauge kinetic term and hence the gaugino mass. Thus, we can summarize all effects by giving the following set of SUSY-breaking parameters and the μ term:

$$m_{1/2} = \frac{\bar{F}_T}{T + \bar{T}} \frac{1}{1 + c'} , \quad (32)$$

$$\begin{aligned} B\mu &= |\mu|^2 + m_{H_u}^2 = |\mu|^2 + m_{H_d}^2 \\ &= |F_\varphi|^2 - \frac{(F_\varphi \bar{F}_T + \text{h.c.})}{T + \bar{T}} \frac{1 + 2c'}{1 + c'} + \frac{|F_T|^2}{(T + \bar{T})^2} \frac{2c'^2}{(1 + c')^2} , \end{aligned} \quad (33)$$

$$\mu = \bar{F}_\varphi - \frac{\bar{F}_T}{T + \bar{T}} \frac{1 + 2c'}{1 + c'} . \quad (34)$$

The most striking feature of this result is, as without the Chern-Simons term, the equality between the $B\mu$ term and the parameters $|\mu|^2 + m_{H_u}^2$ and $|\mu|^2 + m_{H_d}^2$ [6]. We now briefly discuss the phenomenological consequences of this relation:

It is a well-known fact (see e.g. [26]) that electroweak symmetry breaking, i.e. the destabilization of the vacuum with vanishing Higgs expectation values, requires

$$(B\mu)^2 > (|\mu|^2 + m_{H_u}^2)(|\mu|^2 + m_{H_d}^2) . \quad (35)$$

At the same time, positivity of the quadratic part of the scalar potential along the D -flat directions is guaranteed if

$$2(B\mu) < (|\mu|^2 + m_{H_u}^2) + (|\mu|^2 + m_{H_d}^2) . \quad (36)$$

For the parameters that we have found, both inequalities turn into equalities, apparently disfavouring our scenario phenomenologically. However, our previous analysis was performed at a high scale (the GUT scale or the orbifold-GUT compactification scale, which is usually only marginally lower). Thus, our findings are, in fact, very encouraging since even small running effects can easily turn the high-scale equalities into the desired inequalities of Eqs. (35) and (36).

We now discuss in more detail how this running modification of our high-scale relations may occur. The crucial renormalization group equations are

$$16\pi^2 \frac{d}{dt} \mu = \mu [3|y_t|^2 - 3g_2^2] , \quad (37)$$

$$16\pi^2 \frac{d}{dt} (B\mu) = B\mu [3|y_t|^2 - 3g_2^2] + \mu [6a_t \bar{y}_t + 6g_2^2 M_2] , \quad (38)$$

$$16\pi^2 \frac{d}{dt} m_{H_u}^2 = 6|y_t|^2 [m_{H_u}^2 + m_{Q_3}^2 + m_{u_3}^2] + 6|a_t|^2 - 6g_2^2 |M_2|^2 , \quad (39)$$

$$16\pi^2 \frac{d}{dt} m_{H_d}^2 = -6g_2^2 |M_2|^2 , \quad (40)$$

where, except for writing $B\mu$ instead of b , we follow the conventions of [26]. Since, for the purposes of this paper, we are only interested in qualitative features, we have neglected

all Yukawa couplings and trilinear couplings (except those of the top) as well as the U(1) gauge coupling g_1 .

From the above equations we first immediately recognize the well-known fact that, starting with $m_{H_u}^2 = m_{H_d}^2$ at a high scale, one generically finds $m_{H_u}^2 < m_{H_d}^2$ at the electroweak scale, essentially because of the effects of the large top Yukawa coupling. We also see from the formulae at the beginning of this section that both $m_{H_u}^2$ and $m_{H_d}^2$ can easily be negative from the beginning in our setting.

Thus, $(|\mu|^2 + m_{H_u}^2) < (|\mu|^2 + m_{H_d}^2)$ at the low scale and the inequalities of Eqs. (35) and (36) can, in principle, be satisfied simultaneously. Clearly, whether this actually happens depends on the running of μ and $B\mu$ and on their initial values. This depends, in turn, on the fundamental parameters F_T , F_φ and c' of our construction. Furthermore, the running also depends on the soft masses and trilinear couplings in the top quark sector. Since, as we will discuss in more detail in Sect. 6, the matter fields originate in bulk hypermultiplets, the relevant terms come from the superfields expressions [20]

$$\mathcal{L}_{hyp.,4d} \supset \int d^4\theta \varphi \bar{\varphi} \frac{1}{2} (T + \bar{T}) (H^\dagger e^{-2V} H + H^c e^{2V} H^{c\dagger}) + \int d^2\theta \varphi^3 H^c \Phi H + \text{h.c.} \quad (41)$$

Unfortunately, as will again be explained in Sect. 6 referring to the model of [13], realistic Yukawa couplings require many such hypermultiplet terms with non-trivial bulk profiles as well mixing with brane localized charged fields. Thus, we can not simply write down the soft squark masses and trilinear couplings without entering more deeply in the matter sector of our model.

Nevertheless, we see from the above that, using the freedom of choosing F_T , F_φ , c' and of the bulk field localization and bulk-brane mixing in the matter sector, it is very plausible that realistic low-scale SUSY-breaking parameters and μ term can result from our fundamental high-scale formulae, Eqs. (32)–(34). In situations without a Chern-Simons term, a numerical analysis of the running of the relevant parameters has already been performed in Ref. [6], using certain plausible assumptions about soft parameters in the top-quark sector. The authors came to the conclusion that, given the strong high-scale constraints, correct electroweak symmetry breaking is difficult to achieve. They identified the prediction $m_{H_u, H_d}^2 = -m_{1/2}^2$ as one of the main reasons for this difficulty. However, in our model with a Chern-Simons term, precisely this constraint is lifted. In fact, as one can see from Eqs. (32)–(34), the parameters $m_{1/2}^2$ and m_{H_u, H_d}^2 blow up for different negative values of c' , implying that any high-scale ratio of these quantities can, in principle, be realized. Indeed, as has recently been demonstrated in [17], the inclusion of the Chern-Simons term in this type of gauge-Higgs unification models allows for a realistic low-energy phenomenology.

6 An explicit SU(6) orbifold-GUT model

Both the U(6) model analysed above as well as the more minimal pure SU(6) model briefly discussed in a footnote in Sect. 3, do not represent ‘clean’ versions of field-theoretic

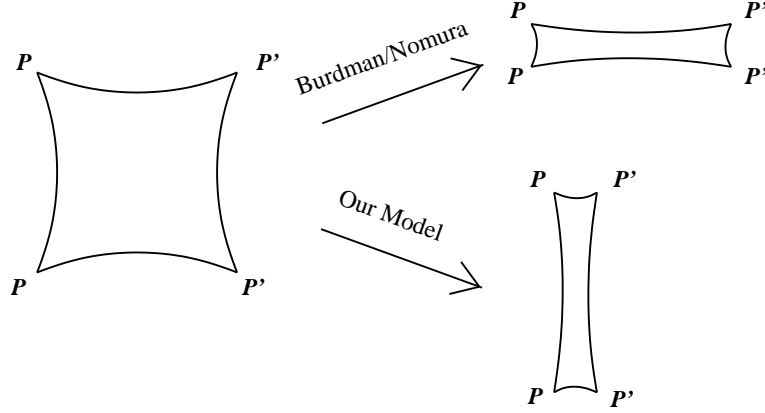


Figure 1: In two different 5d limits, the 6d model described in the text goes over into the model of Burdman/Nomura or into ‘Our Model’.

orbifolding. Indeed, the $U(1)$ factor in $U(6)$ does not allow, in the presence of charged matter, for a breaking by a Z_2 symmetry of the original action. The pure $SU(6)$ model, on the other hand, inherently relies on the gauge symmetry breaking by (non-orbifold) boundary conditions. Thus, it is interesting to see whether a 5d model can be found which realizes all the essential features of our scenario by just modding out a set of Z_2 symmetries. In the present section, we provide a positive answer to this question, modifying the model of [13] appropriately. However, this construction has problems of its own which are related to precision gauge coupling unification (see below).

Although we are ultimately interested in 5d orbifold GUT models with gauge-Higgs unification, the simplest way to approach our model is from a 6d perspective. We start from 6d $\mathcal{N} = 2$ super-Yang-Mills theory with gauge group $SU(6)$ compactified on a torus T^2 . The torus is parameterized by a complex coordinate z with the fundamental domain being defined by $0 \leq \text{Re}z < 2\pi R_6$ and $0 \leq \text{Im}z < 2\pi R_5$. We restrict the field space of the model by requiring invariance under two orbifold projections P and P' . With each of these operations we associate $SU(6)$ matrices which characterize the orbifold action in gauge space and which we denote by the same symbol: $P = i \text{diag}(1, 1, 1, 1, 1, -1)$ and $P' = \text{diag}(1, 1, -1, -1, -1, -1)$. The invariance requirements for the $\mathcal{N} = 1$ vector superfield V contained in the 6d gauge multiplet are

$$PV(z)P^{-1} = V(-z) \quad \text{and} \quad P'V(z - \pi/2)P'^{-1} = V(-(z - \pi/2)). \quad (42)$$

Similar relations, but with an extra minus sign, hold for the chiral superfield Φ , which contains the remaining degrees of freedom of the 6d gauge multiplet.

The resulting theory can be visualized as a 6d model the compactification space of which has the geometry of a pillow (cf. Fig. 1). This space has four conical singularities, each with deficit angle π , two of which are due to the projection P and the other two of which are due to the projection P' . Correspondingly, the gauge symmetry is locally restricted at these singularities to $SU(5) \times U(1)$ for P and to $SU(4) \times SU(2) \times U(1)$ for P' .

We now observe that by taking the limit $R_5 \rightarrow 0$, we arrive precisely at the 5d

orbifold GUT model with gauge-Higgs unification of Burdman and Nomura [13]. This limit is illustrated in Fig. 1. Indeed, in this limit the pillow degenerates to an interval and the fixed points with gauge group $SU(5) \times U(1)$ (labelled by P) merge into a boundary of the 5d space with the same local gauge symmetry. Analogously, the two fixed points with gauge group $SU(4) \times SU(2) \times U(1)$ merge and play the role of the other boundary or brane.

We define our model by keeping R_5 finite and taking the limit $R_6 \rightarrow 0$. This situation, which is also visualized in the figure, corresponds again to a 5d model compactified on an interval. However, the two boundaries are now equivalent and the gauge symmetry at the boundary, which is restricted by both P and P' , is the intersection of the two groups left invariant by the two projections. It is just the gauge symmetry of the standard model plus an extra $U(1)$ factor (the $U(1)$ left over when $SU(6)$ is broken to $SU(5)$).

The model that we have thus obtained is similar but not identical to the 5d model of Sect. 3: The original gauge symmetry, which is $SU(6)$ rather than $U(6)$, is broken at each boundary of the interval to $\mathcal{G}_{SM} \times U(1)$ rather than simply to $SU(5) \times U(1)$. In addition, the vacuum expectation value of Φ takes a less symmetric form. To determine this vacuum expectation value, we first recall that the scalar part of the chiral superfield Φ (which we denote by the same symbol) reads $\Phi = A_6 + iA_5$ in the 6d construction. Furthermore, if a charged particle encircles the stretched pillow (labelled ‘Our model’ in Fig. 1) in the short direction, it experiences a gauge rotation

$$P \cdot P' = \exp[i(\pi/4)T] = \exp \left[i \int_0^{\pi R_6} A_6 dx^6 \right]. \quad (43)$$

Here $T = \text{diag}(1, 1, -1, -1, -1, 1)$ is the generator of the gauge twist $P \cdot P'$ which is felt in the bulk of our effective 5d space and which breaks $SU(6)$ to $SU(3) \times SU(3) \times U(1)$. Thus, after dimensional reduction from 6d to 5d, we find $\langle \Phi \rangle = v \text{diag}(1, 1, -1, -1, -1, 1)$ with $v = 1/(4R_6)$.

This result may appear puzzling since it seems to imply that the physical effects of v , introduced via the Chern-Simons term, become dominant in the 5d limit $R_6 \rightarrow 0$. However, this is not the case for the following reason: The smallest R_6 for which our 6d motivation of the 5d model makes sense is $R_6 \sim g_6$. For smaller R_6 , the 6d approach is compromised by the fact that the strong-coupling scale of the 6d gauge theory lies below the compactification scale. Through the relation $1/g_5^2 \sim R_6/g_6^2$, this limiting situation gives rise to an effective 5d gauge-coupling $g_5 \sim \sqrt{R_6}$. We thus conclude from Eq. (30) that the dimensionless parameter c' governing the size of the physical effects induced by v is indeed $\mathcal{O}(1)$ if the coefficient of the Chern-Simons term in the original lagrangian is $c \sim \mathcal{O}(1)$. Of course, the 6d supersymmetric gauge theory does not allow for a Chern-Simons term. However, the 5d theory obtained after S^1 -compactification includes such a term because of loop effects. The group-theoretic structure of these loop induced Chern-Simons terms, which have been discussed in some detail in Sect. 5 of [16] (see also [14]), is somewhat different from that of the tree-level 5d Chern-Simons term.⁷ However, the

⁷ Such structures are possible because the loop-induced prepotential does not have to be holomorphic at the origin, $\Phi = 0$. This allows for gauge invariant expressions different from $\text{tr } \Phi^3$.

coefficient follows entirely from group theory and matter content and is thus naturally $\mathcal{O}(1)$. We will not derive these terms explicitly in the present 6d-motivated model but only reiterate that, as we claimed before, the physical effects of the Chern-Simons term in the presence of v do indeed arise in more fundamental constructions and are, in general, comparable to the effects derived from the quadratic lagrangian.

Let us finally turn to the problem of standard model matter fields and Yukawa couplings in the presented gauge-Higgs unification model. This is, in principle, a highly non-trivial issue since charged hypermultiplets have to be introduced in the bulk in such a way that, after the orbifold projections, the correct low-energy spectrum results. Furthermore, large 4d Yukawa couplings (in particular that of the top quark) can only result from bulk gauge couplings because the two Higgs doublets come from the chiral superfield Φ in the **35**, which is part of the gauge multiplet and can not have any other interactions in the 5d (or 6d) bulk.

However, concerning all of these issues we can simply refer the reader to the 5d SU(6) model of [13]. In this model, all of the above issues have been solved: For example, the down- and up-type quarks are introduced as hypermultiplets in the **15** and **20** of SU(6) in the bulk, which mix with extra 4d chiral superfields introduced on the branes. It has then been shown that the top- and other Yukawa couplings can be correctly reproduced from the 5d couplings with the gauge multiplet. A similar procedure works for the leptons. The hierarchies of the Yukawa couplings can be realized by allowing for 5d bulk masses for the hypermultiplets, which lead to exponential profiles of the fields and hence to very different effective 4d couplings for the zero modes of the hypermultiplets.

Indeed, the whole construction of [13] can straightforwardly be lifted to 6 dimensions. The field content in 5d and 6d is exactly the same. The orbifold $S^1/(Z_2 \times Z'_2)$ can be replaced by $T^2/(Z_2 \times Z'_2)$, as is visualized in Fig. 1. Instead of placing extra 4d chiral superfields and 4d superpotentials on the boundaries of the 5d interval, those can equally well be placed at the conical singularities of the 6d orbifold. In short, the whole construction goes through without change. A critical issue appears to be the introduction of 5d bulk masses for the hypermultiplets, which is not possible for charged hypermultiplets in 6 dimensions. However, the 6d hypermultiplets may be charged under extra U(1) gauge groups. Wilson lines of these gauge groups (i.e. vacuum expectation values of A_6) then play the same role as 5d bulk masses and lead to localization effects for the zero modes. To summarize, we could simply copy the relevant pages of [13], changing the language from 5d to 6d. We will not do so since, in this paper, we do not intend to go beyond the demonstration that the type of model underlying our discussion of SUSY breaking in the Higgs sector does indeed arise in phenomenologically viable GUT models.

Although the 6d lift of the 5d model of [13] and its ‘opposite’ 5d limit appear to be a very nice motivation of our 5d framework, this is not the only way to approach our construction. Instead, we could simply say that our model is defined, from the start, on a 5d interval with gauge group SU(6) in the bulk. At each boundary, the gauge group is broken to $\mathcal{G}_{\text{SM}} \times \text{U}(1)$ (which is not a Z_2 orbifold breaking) and a non-zero vacuum expectation value for Σ is enforced by the boundary conditions. The inclusion of matter and the generation of Yukawa couplings can be achieved in analogy to the similar

5d gauge-Higgs unification model of [13]. From this perspective, our model remains 5-dimensional. The ‘pillow’ of Fig. 1 and its 5d limit merely serve to convince the reader that non-orbifold 5d boundary conditions are natural, for example as the result of two merging conical singularities with gauge breaking by P and P' .

We finally note that, since the 5d vev used in this section does not preserve the $SU(5)$ subgroup, large threshold corrections to gauge-coupling unification will generically be present [16]. This is not necessarily fatal since the size of these thresholds and the way in which they affect the low-energy couplings is highly model dependent. However, it would require a more detailed analysis to establish whether a fully realistic low-energy phenomenology can emerge. Such an analysis is beyond the scope of the present investigation.

7 Conclusions

We have analysed supersymmetry breaking and the supersymmetric μ term in the Higgs sector of 5-dimensional models with gauge-Higgs unification. This setting is well-motivated both from the perspective of 5d or 6d orbifold GUTs, which are arguably the simplest realistic grand unified theories on the market, as well as from the perspective of the most successful heterotic string models.

Gaugino masses, soft Higgs masses, as well as the μ and $B\mu$ term are generated in a natural way once the F terms of the radion superfield and the chiral compensator acquire non-zero vacuum expectation values. This happens in many of the simplest models where the radion (the size of the 5th dimension) is stabilized with the help of a non-trivial superpotential. The relative size of the SUSY-breaking parameters and the μ term depend on ratio of the two F terms, F_φ/F_T . The overall scale is set by the ratio of the radion F term and the size of the extra dimension, F_T/T . This means that low-scale supersymmetry is realized if the high-scale theory exhibits weak Scherk-Schwarz breaking (known as radion mediation).

In addition to the effects based on the quadratic gauge theory lagrangian, the 5d supersymmetric Chern-Simons term can play a crucial role. This is, in fact, expected since the Chern-Simons term is an unavoidable part of generic 5d models compactified on an interval. Its importance for the low-energy effective theory depends on the presence of a large vacuum expectation value of the 5d scalar in the gauge multiplet. Such a vacuum expectation value can be viewed as a Wilson line from the perspective an underlying 6d or string model. Its size is then naturally of the right order of magnitude to compete with the effects of the quadratic lagrangian.

If, as explained above, supersymmetry breaking is governed by both the quadratic lagrangian and the Chern-Simons term, all relevant terms are generated just on the basis of the F term of the chiral compensator. One can then consider the limit where the F term of the chiral compensator vanishes, corresponding e.g. to the stabilization of the radion purely by Kähler corrections.

The details of the resulting low-energy phenomenology are sensitive to the various high-scale parameters, in particular F_φ , F_T and the vacuum expectation value of the 5d scalar (the real part of the chiral adjoint). However, an interesting feature that appears to be universal within the class of models that we have investigated is the high-scale relation $B\mu = |\mu|^2 + m_{H_u}^2 = |\mu|^2 + m_{H_d}^2$. This relation between $B\mu$ term, μ term and soft Higgs masses is at the borderline of validity of the standard inequalities which have to be imposed for successful electroweak symmetry breaking. Thus, we rely on running effects to lift the equality $m_{H_u}^2 = m_{H_d}^2$, which is standard, and on an appropriate running of μ and $B\mu$ to satisfy the necessary low-energy constraints. As demonstrated in [17], the Chern-Simons term, which lifts certain extra constraints, is crucial to avoid the negative conclusions concerning the low-energy phenomenology of related models reached in [6]. Thus, the proposed version of supersymmetric gauge-Higgs unification with a 5d Chern-Simons term defines an interesting new class of potentially realistic GUT models.

Acknowledgments

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Appendix

This appendix is devoted to the construction of a superfield expression for the non-abelian supersymmetric Chern-Simons term. Suppressing a possible overall prefactor, the superfield expression for the abelian 5d Chern-Simons term is given by [19]

$$\begin{aligned}\mathcal{L}_{cs} &= \int d^2\theta \, \Phi W^2 + \text{h.c.} \\ &+ \frac{2}{3} \int d^4\theta \, (\partial_5 V D_\alpha V - V D_\alpha \partial_5 V) W^\alpha + \text{h.c.} \\ &- \frac{1}{6} \int d^4\theta \, (2\partial_5 V - (\Phi + \bar{\Phi}))^3.\end{aligned}\tag{44}$$

The simple 4d procedure for the non-abelian generalization, i.e. the replacement $V \rightarrow e^{\pm 2V}$, does not work in this case. Instead, we construct the non-abelian lagrangian by matching an appropriate superfield expression (in Wess-Zumino gauge) to the component action. Working within this approach is straightforward because the number of possible superfield actions is highly restricted and the calculation can be performed in close analogy to the abelian case.

Our starting point is the 5d Chern-Simons action of the (non-supersymmetric) non-abelian gauge theory, which can be constructed from the 5d Chern-Simons form given in [31]:

$$\mathcal{L}_{cs\,gauge} = \epsilon^{MNO PQ} \text{tr} \left(\frac{1}{4} A_M F_{NO} F_{PQ} - \frac{i}{4} A_M A_N A_O F_{PQ} - \frac{1}{10} A_M A_N A_O A_P A_Q \right) \tag{45}$$

with the non-abelian field strength

$$F_{MN} = \partial_M A_N - \partial_N A_M + i[A_M, A_N]. \quad (46)$$

This expression must be reproduced by a superfield lagrangian which contains the fields Φ, V, W_α with bosonic components

$$\begin{aligned} \Phi &= \Sigma(y) + iA_5(y) + \theta^2 F_\Phi(y) \\ V_{WZ} &= -\theta \sigma^\mu \bar{\theta} A_\mu(x) + \frac{1}{2} \theta^2 \bar{\theta}^2 D(x) \\ W_\alpha &= \theta_\alpha D(y) - i(\sigma^{\mu\nu})_\alpha{}^\beta \theta_\beta F_{\mu\nu}(y), \end{aligned} \quad (47)$$

where $y = x + i\theta\sigma\bar{\theta}$. Note that the field strength superfield

$$W_\alpha = -\frac{1}{8} \bar{D}^2 (e^{-2V} D_\alpha e^{2V}) \quad (48)$$

gives, in Wess-Zumino gauge, only terms linear and quadratic in V :

$$W_\alpha = W_\alpha^{(1)} + W_\alpha^{(2)} \quad (49)$$

with

$$\begin{aligned} W_\alpha^{(1)} &= -\frac{1}{4} \bar{D}^2 D_\alpha V = \theta_\alpha D(y) - 2i(\sigma^{\mu\nu})_\alpha{}^\beta \theta_\beta \partial_\mu A_\nu(y) \\ W_\alpha^{(2)} &= -\frac{1}{4} \bar{D}^2 [D_\alpha V, V] = 2(\sigma^{\mu\nu})_\alpha{}^\beta \theta_\beta A_\mu(y) A_\nu(y), \end{aligned} \quad (50)$$

which reproduces the expression in Eq. (47).

It is convenient to rewrite Eq. (45) as

$$\mathcal{L}_{cs\ gauge} = \epsilon^{\mu\nu\rho\sigma} \text{tr} \left(\frac{3}{4} A_5 F_{\mu\nu} F_{\rho\sigma} - \frac{1}{2} \{A_\mu, \partial_5 A_\nu\} F_{\rho\sigma} + \frac{i}{4} \{A_\mu, \partial_5 A_\nu\} A_\rho A_\sigma \right), \quad (51)$$

where the curly brackets denote anticommutators. It can be checked that the variation of this expression under gauge transformations is a total derivative.

The first term in Eq. (51) is obtained from a superfield lagrangian which is of the same form as in the abelian case:

$$\text{tr} \left(\int d^2\theta \Phi W^\alpha W_\alpha + \text{h.c.} \right). \quad (52)$$

The second term is reproduced by a piece which is also similar to the abelian case:

$$\text{tr} \left(\int d^4\theta (\{\partial_5 V, D_\alpha V\} - \{V, \partial_5 D_\alpha V\}) W^\alpha + \text{h.c.} \right). \quad (53)$$

For the last term, it is necessary to use just the part of W_α quadratic in V :

$$\text{tr} \left(\int d^4\theta (\{\partial_5 V, D_\alpha V\} - \{V, \partial_5 D_\alpha V\}) W_{(2)}^\alpha + \text{h.c.} \right). \quad (54)$$

The above three terms already reproduce the non-supersymmetric 5d CS term of Eq. (51), but 5d Lorentz invariance is violated by a term $\sim \Sigma F_{\mu\nu} F^{\mu\nu}$ coming from Eq. (52). This can be cured by adding a further contribution, which is a simple generalization of the last term in the abelian CS action:

$$\text{tr} \int d^4\theta \left(e^{-2V} \nabla_5 e^{2V} \right)^3. \quad (55)$$

Here we have used the super gauge covariant derivative

$$\nabla_5 \equiv \partial_5 + \Phi, \quad (56)$$

acting on e^{2V} as

$$\nabla_5 e^{2V} = \partial_5 e^{2V} - \Phi^\dagger e^{2V} - e^{2V} \Phi. \quad (57)$$

The relative prefactors of the four contributions of Eqs. (52)–(55) are fixed by an explicit calculation and found to be consistent with those of the abelian action. Up to an overall constant factor, the result is that of Eq. (11). Although the evaluation of this manifestly supersymmetric expression in WZ gauge reproduces the CS component lagrangian of Eq. (45), we were not able to show that it transforms into a total derivative under super gauge transformations. Most probably this is due to missing extra terms that vanish in WZ gauge. It would be interesting to construct these missing contributions and achieve manifest super gauge invariance (as it is realized for the leading order lagrangian in Eq. (9)).

It requires a certain amount of work to extract even just the bosonic part of our full superfield Chern-Simons lagrangian. One has to integrate by parts using the fact that Σ vanishes at the boundaries. Furthermore, F_Φ is set to zero by the equations of motion, while D takes the value

$$D = -\partial_5 \Sigma + i[\Sigma, A_5]. \quad (58)$$

The final result is

$$\mathcal{L}_{cs} \supset c \left[\frac{2}{3} \mathcal{L}_{cs \text{ gauge}} - \text{tr} \left(\Sigma F_{MN} F^{MN} + 2 \Sigma (D_M \Sigma) (D^M \Sigma) \right) \right], \quad (59)$$

where

$$D_M \Sigma = \partial_M \Sigma + i[A_M, \Sigma]. \quad (60)$$

This also fixes the normalization of our superfield expression relative to the non-supersymmetric Chern-Simons term.

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